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# Study of Some Graph Operators in Domination Theory

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## Abstract:

Graph theory is a fundamental area of discrete mathematics with vast applications in network analysis, computer science, biology, and social sciences. Within this field, domination theory is a critical concept that deals with the selection of a set of vertices such that every vertex in the graph is either in the set or adjacent to a vertex in the set. This paper explores the impact and interaction of various graph operators on domination parameters such as domination number, total domination, independent domination, and power domination. Through examples and theoretical exploration, we analyze how certain graph operations like the complement, line graph, Cartesian product, corona, and join affect domination-related properties.

**Keywords**: Domination in Graphs; Graph Operators, Domination Parameters

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## 1. Introduction

Graph domination theory is a rapidly evolving domain that studies the influence of dominating sets in graphs. A dominating set in a graph $G = (V, E)$ is a subset $D \subseteq V$ such that every vertex not in D is adjacent to at least one member of D. The smallest size of such a set is called the domination number, denoted by $\gamma(G)$. Graph operators, such as the complement, Cartesian product, corona, and line graph, transform a graph into another, often altering its structural properties and, consequently, the domination parameters. Understanding these changes is essential for applications in network design, optimization, and computational biology. In this paper, we explore various operators and examine their influence on domination parameters with proofs and examples.

## 2. Basic Definitions and Notation

Let $G = (V, E)$ be a simple, undirected graph. The following terms are used throughout:

- $\gamma(G)$: Domination number
- $\gamma\_t(G)$: Total domination number
- $\gamma\_i(G)$: Independent domination number
- $\gamma\_p(G)$: Power domination number

Graph Operators considered:

- Complement Graph ($\bar{G}$): A graph on the same vertex set where two vertices are adjacent if and only if they are not adjacent in G.

- Line Graph (L(G)): A graph where each vertex represents an edge of G, and two vertices are adjacent if and only if their corresponding edges share a vertex.

- Cartesian Product $(G \square H)$: A graph with vertex set $V(G) \times V(H)$, where $(u, v)$ is adjacent to $(u', v')\ if\ u = u'\ and\ vv' \in E(H), or\ v = v'\ and\ uu' \in E(G).$

- Corona $(G \odot H)$: For every vertex v ∈ G, attach a copy of H and connect v to every vertex in that copy.

- Join (G + H): Union of G and H, adding edges between every vertex of G and every vertex of H.

## 3. Domination in Complement Graphs

The complement graph can drastically alter domination properties.

Theorem 3.1: If G is a graph of order n, then $\gamma(G) + \gamma(\bar{G}) \leq n.$

Example: $Let\ G\ be\ a\ path\ P_4. Then\ \gamma(P_4) = 2, and\ \gamma(\bar{G}) = 2, hence\ 2 + 2 = 4 = n.$

## 4. Domination in Line Graphs

The domination number of a line graph is closely related to edge domination in the original graph.

Theorem 4.1: $\gamma(L(G)) \leq \gamma'(G), where\ \gamma'(G)\ is\ the\ edge\ domination\ number\ of\ G.$

Example: $For\ a\ star\ graph\ K^1{,}_n, L(K^1{,}_n) = K_n, and\ hence\ \gamma\left(L(K^1{,}_n)\right) = 1.$

**5. Domination in Cartesian Product Graphs**

The Cartesian product graph's domination number is typically bounded below by the domination number of one factor.

**Theorem 5.1:** $\gamma(G \square H) \geq \gamma(G) \cdot \gamma(H).$

**Example:** $G = P_2, H = P_2. Then\ G \square H = C_4, and\ \gamma(C_4) = 2$, which satisfies the bound.

The Cartesian product of two graphs G and H, denoted by $G \square H$, combines the vertex sets of both graphs to create a new graph where adjacency is inherited from both G and H. Formally, the vertex set of $G \square H$ is the Cartesian product $V(G) \times V(H)$. Two vertices (u,v) and $(u', v')\ in\ G \square H$ are adjacent if and only if:

$1. u = u'\ and\ vv' \in E(H), or$

$2. v = v'\ and\ uu' \in E(G).$

The domination number $\gamma(G \square H)$ refers to the minimum number of vertices required in a dominating set such that every vertex in $G \square H$ is either in the set or adjacent to a vertex in the set.

**5.1Vizing's Conjecture**

One of the most well-known conjectures in this area is Vizing's Conjecture:

$\gamma(G \square H) \geq \gamma(G) \cdot \gamma(H)$

This inequality is believed to hold for all graphs G and H, though it remains unproven in the general case.

**Example**

Let $G = P_2$ (a path on 2 vertices) and $H = P_2$. Then $G \square H$ is isomorphic to $C_4$ (a cycle on 4 vertices).

We know:

$\gamma(P_2) = 1$

$\gamma(C_4) = 2$

Thus, $\gamma(G \square H) = \gamma(C_4) = 2 \geq 1 \times 1$, satisfying Vizing's conjecture.

**5.2. Observations**

In general, the domination number of the Cartesian product graph $G\square H$ does not coincide with the product $\gamma(G)\gamma(H)$, highlighting the non-multiplicative nature of domination under this graph operation.

1. Identifying dominating sets in Cartesian product graphs typically demands tailored construction techniques, as straightforward combinations of dominating sets from the factor graphs are often insufficient.
2. Even when the constituent graphs are structurally simple, such as paths or cycles, the determination of the domination number of their Cartesian product becomes significantly more intricate.

### 5.3. Applications

Understanding domination in Cartesian products is important for:
- Grid networks (e.g., sensor placements)
- Image processing (pixels as graph nodes)
- Parallel computing and interconnection networks

## 6. Domination in Corona Graphs

The corona operation greatly increases the domination number.
Theorem 6.1: If H is non-null, then $\gamma(G \odot H) = |V(G)|$.
Example: $For\ G = P_2, H = K_1, we\ have\ \gamma(G \odot H) = 2.$

## 7. Domination in Join Graphs

The join operation typically reduces the domination number.
Theorem 7.1: $\gamma(G + H) = 1$ if neither G nor H is empty.
Example: $For\ G = K_2, H = K_3, G + H\ is\ complete, hence\ \gamma(G + H) = 1.$

## 8. Power Domination

Power domination, motivated by monitoring electrical networks, differs from standard domination.

Theorem 8.1: $\gamma_p(G) \leq \gamma(G)$, since power domination allows for propagation.

Example: In a path $P_4$, $\gamma(P_4) = 2, but\ \gamma p(P_4) = 1.$

## 9. Conclusion

Graph operators play a significant role in altering domination parameters, which has strong implications in theoretical research and practical applications. This paper reviewed the effect of standard operators on various domination parameters, supported by examples and known results. Future work may involve computational approaches to estimate domination parameters under more complex graph operations or in dynamic graphs.

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